Write your name here:

SECTION A Maximum 60	SECTION B Maximum 24		TOTAL	

CHRIST'S HOSPITAL

ENTRANCE EXAMINATION PAPER: YEAR 9

Year 8 students, for admission to Year 9 in September 2014

Mathematics

Time allowed: 60 minutes

There are two Sections, A and B.

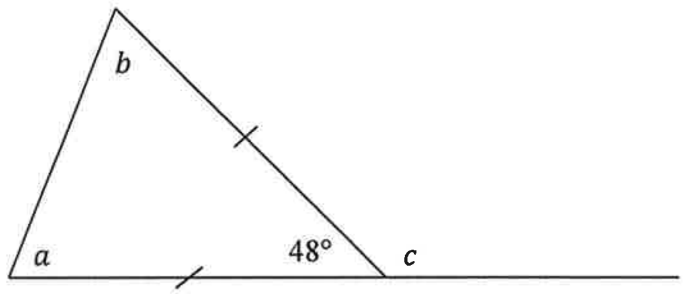
You are advised to spend about 40 minutes on Section A and 20 minutes on Section B. Do not worry if you find that Section B is rather different from the kind of mathematics questions you have done before; it tests your comprehension skills.

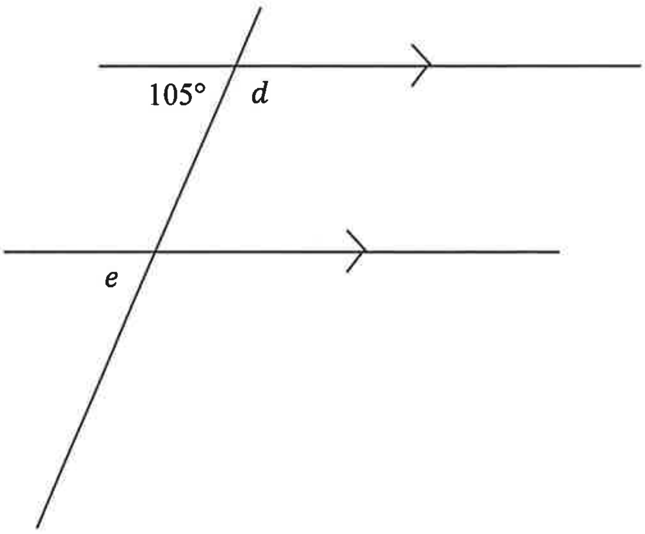
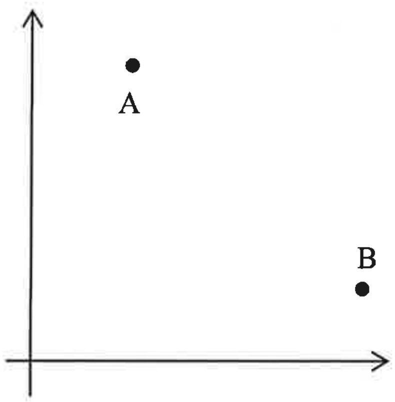
1. Write your answers clearly.
2. Working should be shown in the spaces provided, where appropriate.
3. If your answer is wrong but you have shown some working then you may still earn some "method" marks.
4. Calculators must NOT be used.

SECTION A

1	a) Write the number 18.0663 correct to 2 decimal places.	[1]
	b) Write the number 10.234 correct to 3 significant figures.	[1]
2	Find two numbers that add up to give −12 and multiply together to make 32.	[1]
3	Write $8 \times 8 \times 8 \times 8$ using power (index) notation.	[1]
4	Find the Lowest Common Multiple (LCM) of 24 and 30.	[1]

5	"If the Highest Common Factor of two whole numbers is 1, then each of those two numbers must be a prime number." Is this statement TRUE or FALSE?	**[1]**
6	Which one of these is closest in value to 7? $\sqrt{35}$ $\sqrt{40}$ $\sqrt{45}$ $\sqrt{50}$ $\sqrt{55}$	**[1]**
7	Write 360 as a product of its prime factors. (You may wish to use a factor tree.)	**[2]**

8	Fill in the missing numbers to make a regular sequence: 21 3	**[2]**
9	Write a rule for the n-th term of the sequence $9, 14, 19, 24, 29, \ldots$	**[2]**
10	Find the values of the angles marked a, b, c.  $a =$ ...., $b =$ ...., $c =$	**[3]**

11	Find the values of the angles marked d and e.  $d =$ ...., $e =$	**[2]**
12	A line is drawn from the point A (1, 6) to the point B (9, 2). M is the midpoint of AB. Find the coordinates of M. 	**[2]**

13 A machine produces large numbers of toy coins, in 5, 10, 20 and 50 pence values.
The machine does not produce equal numbers of each value.
A randomly chosen coin made by the machine will have a value as shown on the table below.
One item of information is missing from the table.

Value, pence	5	10	20	50
Probability	0.4	0.3	0.2	

One coin is chosen at random. Find the probability that it has a value of:

a) 50

.... **[1]**

b) 10 or 20

.... **[1]**

c) not 5

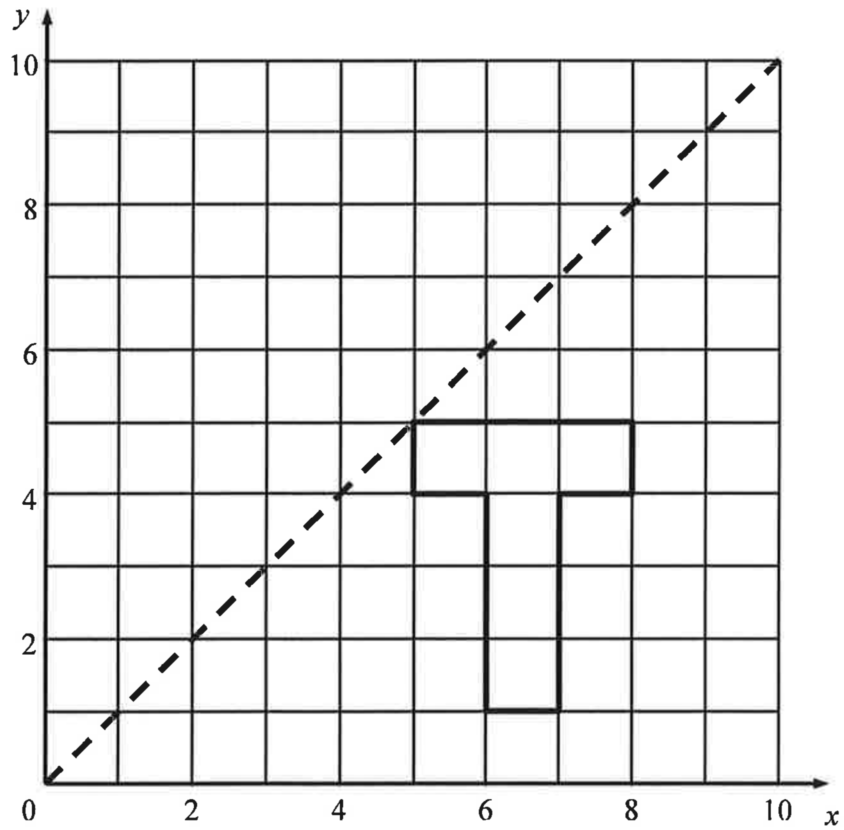
.... **[1]**

d) 40 coins from this machine are chosen at random.
Work out an estimate for the number of 10 pence coins chosen.

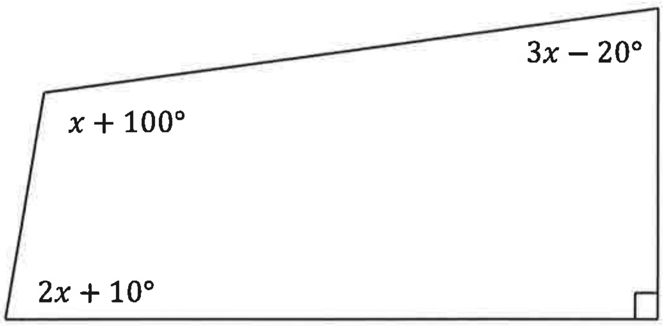
.... **[1]**

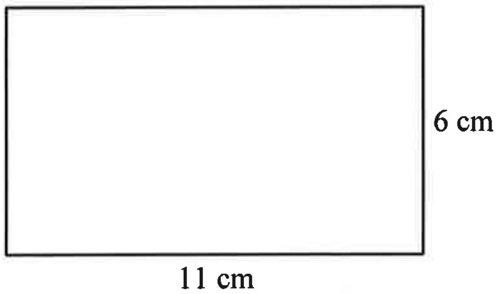
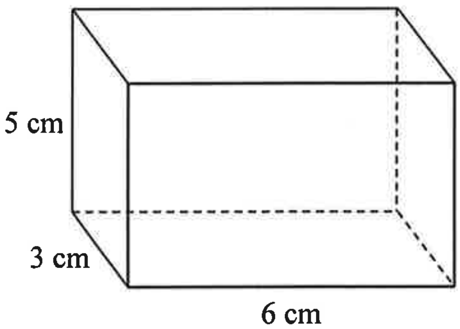
14	Work out $$\frac{2}{3}+\frac{1}{4}$$ Show all of your working clearly.	**[2]**
15	Find 95% of 600.	**[2]**

16	Express 11 as a percentage of 20.	**[2]**
17	Work out $$7\frac{1}{2} - 2\frac{2}{3}$$ Show all of your working clearly, and give your answer as a mixed fraction.	**[2]**

18	Alan wishes to decrease £140 by 10%. Which **one** of these is a correct calculation to achieve this? Put a **tick** ✓ alongside the correct calculation. A 140×0.1 B 140×0.9 C $140 \div 1.1$ D 140×1.1	**[1]**
19	Draw the reflection of the letter T shape in the given mirror line. 	**[1]**

20	Expand and simplify: a) $6(4x + 3)$	**[1]**
	b) $2(x + 7) + 5(2x - 3)$	**[2]**
	c) $10x - 2(x + 3)$	**[2]**
	Copy and complete, filling in the two gaps represented by the blocks □: d) $\square(5x + \square) = 30x + 12$	**[1]**
21	Solve the equation $$5x + 7 = 2x + 31$$ You **must** show your working.	**[2]**

22	Here is a quadrilateral. It is not drawn to scale.  a) Find the value of x.	**[4]**
	b) Work out the value of the largest angle in the quadrilateral.	**[1]**
23	One of these points lies on the line with equation $y = 3x + 1$. Which one is it? (1, 3) (2, 8) (3, 9) (4, 13) (5, 14)	**[1]**

24	a) Find the area of this rectangle. State the units. 	**[2]**
	b) Find the volume of this cuboid. State the units. 	**[2]**
25	Here is a number machine. INPUT x → MULTIPLY BY 2 → SUBTRACT 3 → OUTPUT y a) Find the output y if the input x is 12.	**[1]**
	b) Use algebra to write a rule in the form $y =$ .... $y =$	**[2]**

26	Here are the ages, in years, of 8 adults: 21, 21, 21, 22, 22, 24, 25, 28 Their ages add up to a total of 184 years. a) Work out the mean of their ages. .... b) Write down the mode of their ages. .... c) Work out the median of their ages.	 **[1]** **[1]** **[1]**
27	You are given that $247 \times 43 = 10621$ a) Use this information to work out 2.47×4.3 .... b) Use this information to work out $106210 \div 4300$	 **[1]** **[1]**

END OF SECTION A

SECTION B: Sequences and series

Introducing sequences

A **sequence** is a set of numbers, usually written as a list of items separated by commas. The items are more correctly known as **terms**. Some sequences have a fixed number of terms; these are called **finite sequences**. Other sequences go on forever; they are said to be **infinite sequences**.

Here is an example of a finite sequence:

2, 5, 8, 11, 14

- It has 5 terms
- The first term is 2
- The last term is 14

Now, here is an example of an infinite sequence:

3, 5, 7, 9, 11, ...

- It has infinitely many terms
- The first term is 3
- There is no last term

EXERCISE 1

Find the next two terms in each of these number sequences:

A 12, 15, 18, 21, 24, □, □

B 1, 3, 6, 10, □, □

C 2, 8, 18, 32, 50, □, □

D 1, 8, 27, 64, □, □

4 marks

EXERCISE 2

This exercise refers to the infinite number sequence: 5, 8, 11, 14, 17, ...

E Find the 8^{th} term in the sequence

F Decide whether 103 is a term in this sequence

G Find the 50^{th} term in the sequence

3 marks

Sequences and series

If you add up the terms in a sequence then the result is a **series**, so a series is simply the **sum** of a sequence. For example, 1, 3, 5, 7, 9 is a sequence but 1 + 3 + 5 + 7 + 9 is a series. If this had been part of a longer series then we would say that this is the sum of the first five terms.

EXERCISE 3
Find the sum of these three series:

H 301 + 303 + 305 + 307 + 309

I 5 + 7 + 9 + ... (to 10 terms)

J 1 – 2 + 4 – 8 + 16 – ... (to 12 terms)

3 marks

Arithmetic progressions

A series in which each term differs from the previous one by a constant amount is called an **arithmetic progression**. For example 1, 3, 5, 7, 9 is an arithmetic progression but 1, 3, 6, 10, 15 is not.

EXERCISE 4
Decide whether each of these is an arithmetic progression or not. Just write "Yes" or "No" for each.

K 100, 103, 106, 109, 112, ...

L 100, 200, 400, 800, 1600, ...

M 100, 91, 82, 73, 64, ...

3 marks

Arithmetic progressions and Carl Friedrich Gauss

The German mathematician Carl Friedrich Gauss is credited with inventing a clever way of finding the sum of an arithmetic progression. His teacher asked the class to work out the value of the series 1 + 2 + 3 + ... + 1000. "You have one hour!"

Gauss immediately replied that the answer was 500500. This is what he did.

Take the first and last terms and add them: 1 + 1000 = 1001

Now take the next pair and do the same: 2 + 999 = 1001

Similarly for the next pair: 3 + 998 = 1001

Continuing in this way, you can make pairs of numbers that always add up to 1001. Since there were 1000 numbers to begin with, there must be 500 pairs. So the total is 1001 x 500 = 500500.

EXERCISE 5

N Use the method of Gauss to work out the value of 1 + 3 + 5 + 7 + ... + 499

4 marks

EXERCISE 6

Explain briefly why it would not be possible to use the method of Gauss to work out the value of 1 + 2 + 4 + 8 + ... +1024

2 marks

EXERCISE 7

P Now imagine you are writing a mathematics textbook, and you need to finish writing a set of questions about completing number sequences (similar to Exercise 1 in this task). Suggest 5 interesting number sequences in the spaces below.

There is one important rule – you are only allowed to have **at most one arithmetic progression** in your set of 5 sequences.

1

2

3

4

5

5 marks

END OF SECTION B